Multi-Input Multi-Output Automatic Design Synthesis for Performance and Robustness

Vernon C. Gordon* and D.J. Collins†
Naval Postgraduate School, Monterey, California

A direct-design method for solving the problem of robustness to cross-coupling perturbations in multivariable control systems is presented. The method uses numerical optimization procedures to manipulate the system feedback gains as direct design variables. The manipulation is accomplished in a manner that produces desired performance by pole placement and robustness by modification of the minimum singular values of the system return difference matrix. Channels affected by cross-coupling perturbation may be recognized by the character of their transfer function plots. The mechanism used by the pole placement and robustness routine in obtaining a robust design is evident from the gain changes associated with the Bode diagrams and the zero shifts shown on pole-zero plots. The pole placement and robustness routine uses gain equalization and zero assignment to modify the characteristics of the system in the design. A modification of the pole placement and robustness routine that may be applied to the design of robust observers is also presented. Using feedback and filter gains as direct design variables, a practical design procedure for robustness recovery in observer-based systems is obtained.

Introduction

WITH the rising interest in multivariable control theory brought on by increasingly complex systems, the need has arisen to develop methods that will allow the designer to specify system performance while at the same time ensure relatively high stability margins or robustness. In the single-input, single-output (SISO) case, the designer has had the tools to do these tradeoffs in the form of Nyquist, Bode, and root-locus analysis. In the multi-input, multi-output (MIMO) case, the classical methods are not totally appropriate or adequate.

Numerous techniques have been developed to analyze MIMO systems. Among the methods are decoupled system analysis, diagonally dominant systems and the Linear Quadratic (LQ) method. The robustness of each of these methods can be questionable, particularly to cross-coupling perturbations between loops.

The technique discussed in this paper incorporates time-domain pole-placement design procedures with a method of using the return difference matrix singular values to improve the robustness of MIMO design. The technique, which uses modern numerical optimization routines, can assist the designer in obtaining robustness in the face of cross-coupling perturbations. This pole-placement and robustness design routine has been applied to several problems discussed in recent literature. The pole-placement and robustness design code has proven capable of meeting the desired goals of pole placement and robustness and brought to light some additional aspects of cross-coupling perturbation problems. A modified pole-placement and robustness routine has proven effective in the design of robust observers without large feedback gains.

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*Commander, USN.
†Professor, Aeronautics. Associate Fellow AIAA.

Design Procedure

The task of designing a control system with acceptable time-domain performance and robustness characteristics can be accomplished in a straightforward manner by using a numerical optimization techniques. Numerical procedures are used to vary selected design variables, in this case the feedback gains, to obain a desired performance level. The performance level is a combination of time-domain performance and robustness or frequency-domin performance. The criteria for system performance are established in terms of optimization objective and constraint functions. This provides a versatile method to establish system feedback gains and effect an acceptable design in terms of performance and robustness. The pole placement is used to establish system performance and the minimum singular value to establish robustness. Since, as will be seen, the establishment of robustness results in zero shifts, some iteration may be necessary in determining an adequate design.

The pole-placement portion of the procedure was chosen, because it is relatively easy to implement through numerical optimization routines. The numerical formulation also makes it simple to incorporate robustness into the method along with performance. A numerical technique similar to one posed by Shapiro[1] was chosen for pole placement. The implementation was accomplished using a new optimizer program called the Automatic Design Synthesis (ADS) program, developed by Vanderplaats.[2] This on-demand optimizer provides a versatile method to manipulate pole locations via objective function, by constraint function, or by a combination of both. The program objective function is implemented as Eq. (1).

$$\text{OBJ.} = \Sigma \{\lambda_{R_{D_i}} - \lambda_{R_i}\}^2 + \{\lambda_{I_{D_i}} - \lambda_{I_i}\}^2 \tag{1}$$

where λ_R = real part eigenvalue, λ_I = imaginary part eigenvalue, λ_{R_D} = desired real part eigenvalue, and λ_I = desired imaginary part eigenvalue.

The constraint formulation is written as Eq. (2)

$$g(i) = [\{\lambda_{R_{D_i}} + \lambda_{R_i}\}^2 + \{\lambda_{I_{D_i}} - \lambda_{I_i}\}^2]^{1/2} - r \tag{2}$$

where r is a tolerance circle established as a function of pole placement position.

The pole placement and robustness design program is based on the ADS code and consists of two separate programs. The first program is used to provide designs for state or output feedback problems, while the second program is used for observer or filter designs.

The frequency domain or robustness portion of the design procedure was formulated in a similar manner to that of Mukhopadhyay and Newsom.[3] In this formulation, the objective function is given as Eq. (3).

$$J=\sum_{\omega}\left[\max\left(\sigma_d-\underline{\sigma}\{j\omega,p\}\right)\right]^2 \tag{3}$$

where the σ_d is determined by using the Universal Gain and Phase Singular value plot.[3,4] This formulation is used to keep the return difference singular value above a minimum value and thus provides the desired phase and gain performance as found in Fig. 1.

For the state or output feedback design program, the user must input the plant matrices A, B, C and the initial starting values of the feedback matrix F. The matrices correspond to the following linear differential system:

$$\dot{x}=Ax+Bu \tag{4}$$

$$y=Cx \tag{5}$$

$$u=-Fx \tag{6}$$

Although a feed-forward matrix D could be added to the procedure if required for a specific design case, it has not been considered in the current program.

As the design program is currently coded, the user may run output feedback or state variable feedback by specifying the C matrix as the identity matrix. The program relies on initial starting values of the feedback gains, F. There is no guarantee that the optimum found by the procedure is the global optimum or that the procedure will always converge to an acceptable solution. The ability to select acceptable starting values for the feedback gains will make the routine more efficient in operation. As currently employed, the progam is used to obtain pole placement and robustness for a given set of starting gains and a selected optimization routine from the ADS program. If the optimizer is not able to meet the desired design goals of the program, two options are available. First, change to a different optimization routine from the list of available ADS routines and rerun the problem. This was usually successful in improving the design. Second, the designer selects a new set of starting values for the feedback gains and repeats the design procedure. Both options may be necessary in particularly difficult cases.

The ADS code was developed as a follow-on to the successful CONMIN code also developed by Vanderplaats. It is designed as a black-box optimizer which allows the user to choose combinations of one-dimensional search, optimization algorithm, and optimization strategy. First, forward finite-difference gradients are calculated in the program, but analytical gradients, if available, can be used within the code.

Two strategies used most often in the analysis for this paper have been sequential unconstrained minimization, using quadratic exterior penalty function, and the augmented Lagrange multiplier (ALM) method. Other strategies available include sequential linear programming and sequential quadratic programming.

The basic optimizer is also chosen by the user from two unconstrained and three constrained optimization algorithms. The algorithms are Fletcher-Reeves conjugate directions, Davidon-Fletcher-Powell (DFP) variable metric method and the Broydon-Fletcher-Goldfarb-Shanno (BFGS) variable metric method. The method of feasible directions and robust feasible directions are available for constrained minimization.

The user has available several types of one-dimensional searches using Golden Section or polynomial approximation techniques. The ADS code has tailored these one-dimensional search algorithms for the unconstrained and constrained cases, allowing the user to make appropriate choices for the type of problem to be solved.

The pole-placement and robustness design procedure has consistently been able to find improved designs; however, the program does not always yield acceptable designs. Using the IBM 3033 time-share system, the routine requires about 10 CPU s to work a second-order problem and on the order of 15 to 60 CPU s to run a fourth-order problem.

The observer or filter equations are:

$$\dot{\tilde{x}}=A\tilde{x}+Bu+K[y-C\tilde{x}]$$

$$y=Cx$$

$$u=-F\tilde{x} \tag{7}$$

where x is the state, $\tilde{x}$ the estimator variables, F the feedback gains, and K the observer gains. The designs of the feedback gains and the observer gains are accomplished as separate operations, in keeping with the separation principle. In using the pole placement and robustness design procedure for the observer system, initial values of the F and K matrices must be input. The same or different optimization techniques from ADS may be employed.

The observer robustness design program requires two passes of the ADS program. In the first pass, the feedback gains, F,

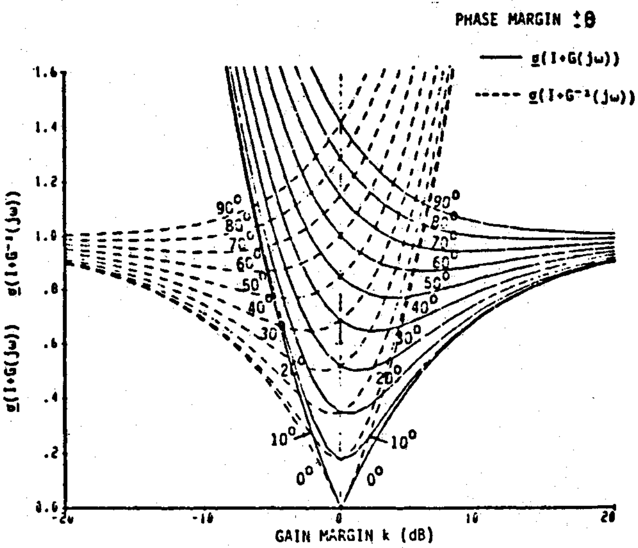

Fig. 1 Universal gain and phase curves.

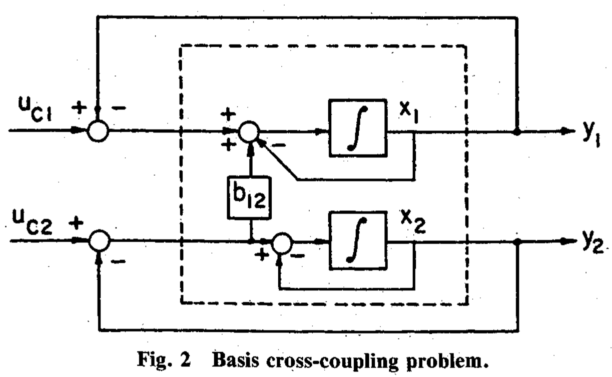

Fig. 2 Basis cross-coupling problem.

of the controller are computed to obtain the desired pole locations for the plant. The second pass of the routine is used to adjust the observer gains to recover the system robustness. The two-pass procedure would need to be modified in the case of a reduced-order observer, since changes to the estimator gains K in this case would change the pole locations of the plant.

The results contained in this paper have been based on the input additive singular value level. One can easily break the loop at other points and also use weighted averages of several break points. Further details of the method and a program listing may be found in Gordon.[5]

Introductory Problem

An almost classical second-order problem is used to demonstrate the ability of the robustness design routine to solve cross-coupling robust design problems. Figure 2 is a diagram of this basic system. In this problem a simple plant is specified by the following linear system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{8}$$

where b_{12} has a large value ($b_{12}=50$). Feedback compensation is chosen as:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} \tag{9}$$

and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{10}$$

The closed-loop system is then:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & b_{12} \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} \tag{11}$$

The system is stable with eigenvalues at -2, -2. We will consider Eq. (11) as a new open-loop system. The transfer matrix for the system is

$$G = C(sI-A)^{-1}B \tag{12}$$

and

$$G = 1/(s+1) \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix} \tag{13}$$

with a return difference matrix of

$$I+G = 1/(s+1) \begin{bmatrix} s+2 & b_{12} \\ 0 & s+2 \end{bmatrix} \tag{14}$$

The inadequacies of classical methods (Nyquist, diagonal system) in establishing the robustness of the system are discussed by Lehtomaki.[6]

The pole-placement and robustness design program was applied to this problem. Pole locations of -2 and -2 were chosen. Using the pole placement algorithm with one variable feedback gain in each loop, the program produced a design with eigenvalues at the desired location (-2, -2). The feedback gains were found to be $F_{11}=1.00$ and $F_{22}=1.00$. The other gains F_{12}, F_{21} were held fixed at zero.

A singular-value plot for the system is shown in Fig. 3. The minimum singular value was on the order of 0.07 or -22.5 dB, indicating poor robustness. A small perturbation from input 1 (u_{c1}) to input 2 (u_{c2}) can cause the system to become unstable. The singular-value plot of the system reflects this physical fact. Pole-zero plots of this case are shown in Fig. 4(a). A third degree of freedom (F_{12}) in the feedback gain selection permitted the optimizer routine to raise the singular value of the system to a specified level (above 0.8) and still place the poles at -2, -2. The optimized singular-value plot is shown in Fig. 4(b). The diagonal feedback gains remained at 1, but the off-diagonal gain was calculated to be $F_{12}=51.83$. This large gain is used to offset the cross-coupling terms in the original problem and increase the robustness of the system to cross-coupling perturbations. This effect can be seen graphically in the comparison given in Fig. 5.

The open-loop Bode plot of the cross-coupled channel [Eq. (11)] shows high gain (35 dB) in the original system and a bandwidth of close to 50 rad/s. A large reduction in the gain and bandwidth is brought about by the additional feedback gain computed in the robustness improvement portion of the design program. The two diagonal channels remain essentially unchanged in the problem. A plot of the new pole-zero configuration is found in Fig. 6. In the figure note the shift of a zero in the off-diagonal plot toward the location of the minimum singular value. Using all four feedback gains as variables produced similar trends for the computed feedback gains.

The robustness problem for this system exists in the upper cross-coupling channel (input 2, output 1). The lack of robustness can be determined in two ways. The first method is to plot the open-loop Bode contour of each element of the transfer matrix. This method gives physical insight into the nature of the problem. The second method examines the singular values of the return-difference matrix. Low singular values correspond to low robustness. The pole-placement and robustness design routine, by modifying feedback gains, automatically reduces the cross-coupling effects and provides a robust system with desired eigenvalues. This gain change is evident in a modification to the open-loop Bode gain and bandwidths and a zero shift toward the minimum singular value. The optimizer routine behaves as if it inserted a lag compensator into the high-gain, large-bandwidth channel.

Helicopter Problem

Three highly coupled two-input system models for the CH-47 have been discussed by Sandell et al.[7] Two of the three designs, while meeting basic performance criteria, had poor robustness. The robustness design procedure was able to provide improvement to both designs.

The model for the CH-47 helicopter was given by:

$$\dot{x} = Ax + Bu$$
$$x^T = (v, p, r, \mu)$$
$$u = (\delta_B, \delta_C) \tag{15}$$

where v is the side slip velocity, p the roll angular rate, r the yaw angular rate, μ the roll angle, and u is the control inputs, with:

$$A = \begin{bmatrix} -2.27 & -1.42 & -0.15 & 31.99 \\ 0.01 & -0.7 & -0.07 & 0.0 \\ 0.04 & -0.05 & -0.05 & 0.0 \\ 0.0 & 1.0 & 0.11 & 0.0 \end{bmatrix} \tag{16}$$

$$B = \begin{bmatrix} 0.12 & 0.95 \\ 0.04 & -8.37 \\ 0.34 & 0.02 \\ 0.0 & 0.0 \end{bmatrix} \tag{17}$$

with full state feedback available.

The control laws are formulated as:

$$u = -F_i x + h_i \mu_c \tag{18}$$

where μ_c is a step input that μ must track and where for each design the F_i and h_i matrices are given by:

$$F_1 = \begin{bmatrix} -1.72 & -23.5 & 70.6 & 595.0 \\ 0.024 & -2.71 & 0.368 & -7.99 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0.198 & 154.0 & 18.3 & 142.0 \\ -0.01 & -1.59 & -0.189 & -1.47 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 0 & 0 & 25.5 & 0 \\ 0 & -4 & 0 & -27 \end{bmatrix} \tag{19}$$

$$h_1 = \begin{bmatrix} 595.0 \\ -7.99 \end{bmatrix}$$

$$h_2 = \begin{bmatrix} 142.0 \\ -1.47 \end{bmatrix}$$

$$h_3 = \begin{bmatrix} 0 \\ -27 \end{bmatrix} \tag{20a}$$

The eigenvalues for each design were determined from the equation

$$|A - BF_i| = 0, \qquad i = 1,2,3 \tag{20b}$$

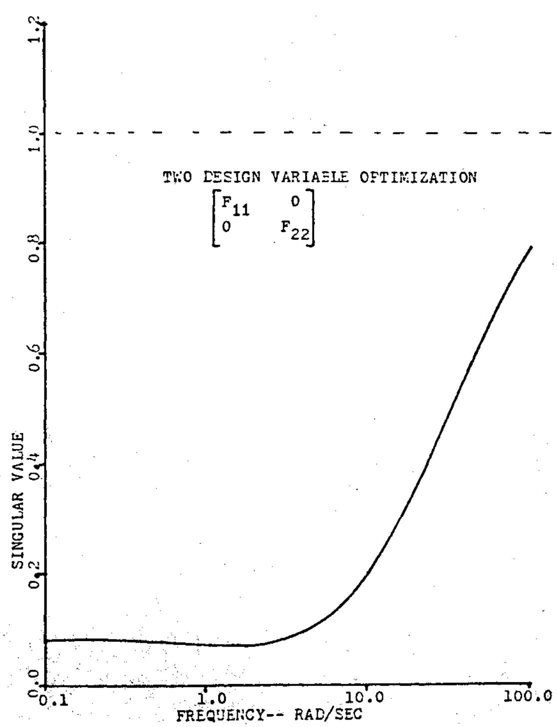

Fig. 3 Singular values for basic problem.

The robustness of these designs can be assessed by computing singular values. This computation shows two of the three designs to have low robustness as can be seen in Fig. 7. Design 1 has singular values of about -20 dB near 10 rad/s, while design 2 has singular values of -34 dB over a wide range of frequency. Design 3 has singular values greater than 1 and is robust.

The pole placement and robustness design technique was applied to the two nonrobust designs, using the eigenvalue locations of each design as indicated above as the desired

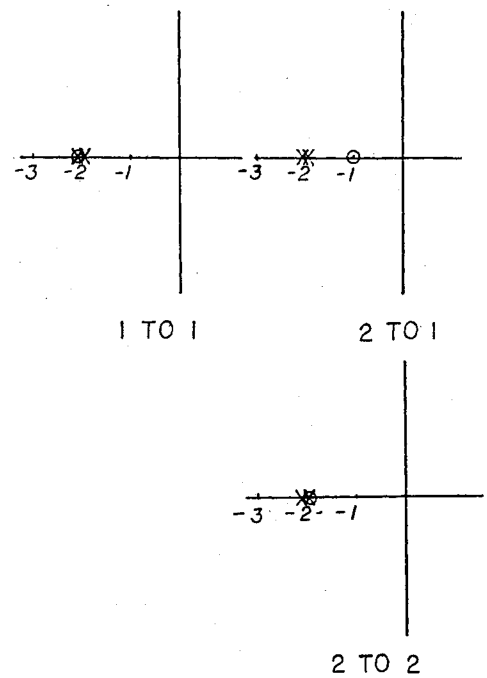

Fig. 4a Pole placement only CL pole-zero plot.

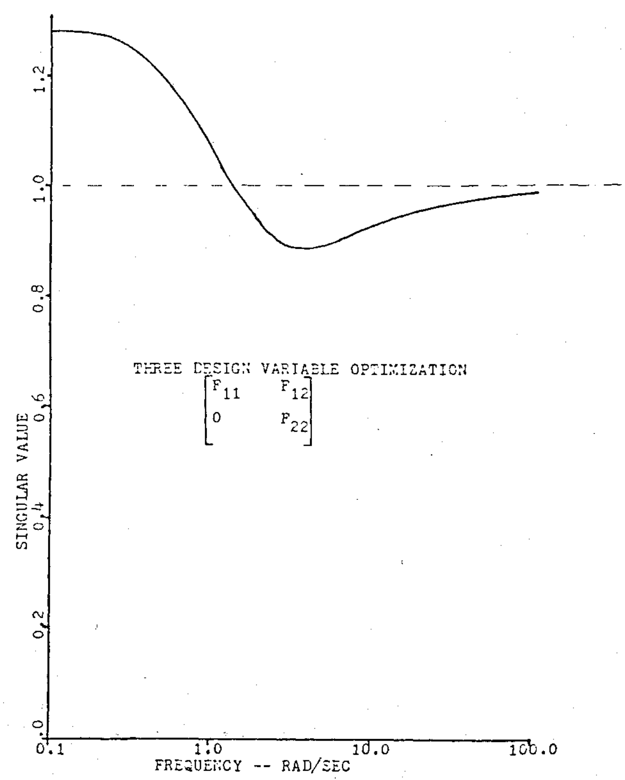

Fig. 4b Singular values optimized basic problem.

pole-placement location. In the first case, cross-feed perturbation through the actuator can produce instability, as indicated by low singular values at lower frequencies. Using a singular value of 0.6 as a desired robustness level, the optimizer program was employed to compute feedback gains for pole placement and increased robustness. The minimum singular value (Fig. 7) was raised from 0.11 to about 0.61. The improvement came from a modification of the feedback gains in channel δ_B.

By greatly reducing the gains in channel δ_B, the optimizer routine minimized the influence of the cross-coupling from channel δ_C. In this way, a much larger spill-over of channel δ_C may be tolerated through the actuator before the system becomes unstable. In the loop Bode plot of input δ_C (2) to output δ_B (1), some important aspects of the problem are observed. This diagram clearly indicates the cross-coupling problem and the mechanism used to optimize for increased robustness. The gain is reduced from 95 to 88 dB and the bandwidth from 130 to 65 rad/s. Figure 8 depicts this change. Channel $\delta_c(2)$ to $\delta_b(1)$ is the prime destabilizing channel in this system, and the optimizer has used modified feedback gain to bring the entire system to more balanced conditions and recover a highly robust design.

The gain changes also cause zeros of the various closed-loop, pole-zero diagrams to move. The optimized design zeros are moved in a direction that attempts to equalize or balance the frequency response for frequencies in the vicinity of the minimum singular values. Figures 9 and 10 show an example of how the pole-zero shifts tend to balance the overall frequency response of the system. The zeros shift into the region, which has a natural frequency of 10, i.e., where the singular value has its minimum. Similar improvement in robustness and changes to transfer function plots and pole-zero diagrams were found for the other cases studied.

The dynamic response of the robust system in design I was essentially the same as the original nonrobust design.

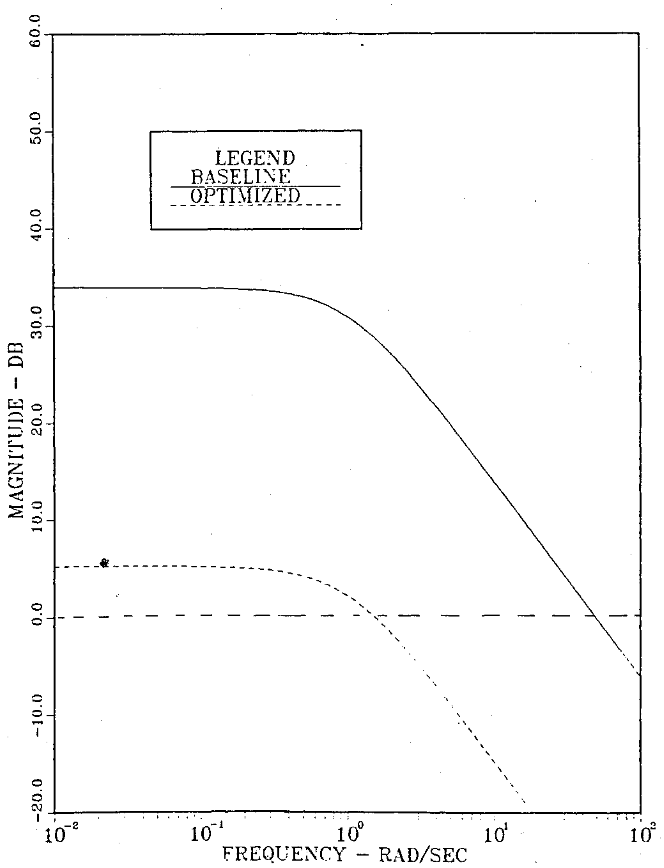

Fig. 5 Bode transfer 2-1.

Simple Observer

The design procedure may be easily adapted to the recovery of robustness in observers. Consider the simple observer[8] developed from the system

$$\dot{x}=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}x\begin{bmatrix}0\\ 1\end{bmatrix}u\begin{bmatrix}35\\ -61\end{bmatrix}\tau$$

$$y=[2\ \ 1]x+\eta$$

$$E(\eta)=E(\tau)=0,\quad E(\eta\eta)=E(\tau\tau)=\delta(t-\tau) \tag{21}$$

Figure 11 is a singular-value plot of the full-state system and the optimal observer. The low singular values for the optimal observer indicate poor gain and phase margins. Nyquist plots of the full-state optimal filter and a fast filter are given by Doyle and Stein.[8] Their procedure of introducing a fictitious process noise leads to robustness recovery but with high filter gains. Using the pole-placement and robustness routine Poplar, one can obtain by direct modification of the filter feedback gains a robust system with moderate gains. The filter poles were constrained to be greater than -2. Without this constraint, one obtained the curve called POPLAR II.

$$k_1=1.0018 \qquad -k_2=2.02742$$

$$\mathrm{GM}=-5.5\ \mathrm{dB} \qquad \mathrm{PM}=117\ \mathrm{deg}$$

The state covariance matrix $E(xx^t)$ is:

$$\begin{bmatrix}306.5 & -456.5\\ -456.5 & 700.1\end{bmatrix}$$

These values compare favorably with the values obtained by the other method. The response curves to a step input match the full state response and are better than the optimal observer. In short, the present design procedure is a highly practical and direct method of obtaining robustness recovery with observers or filters.

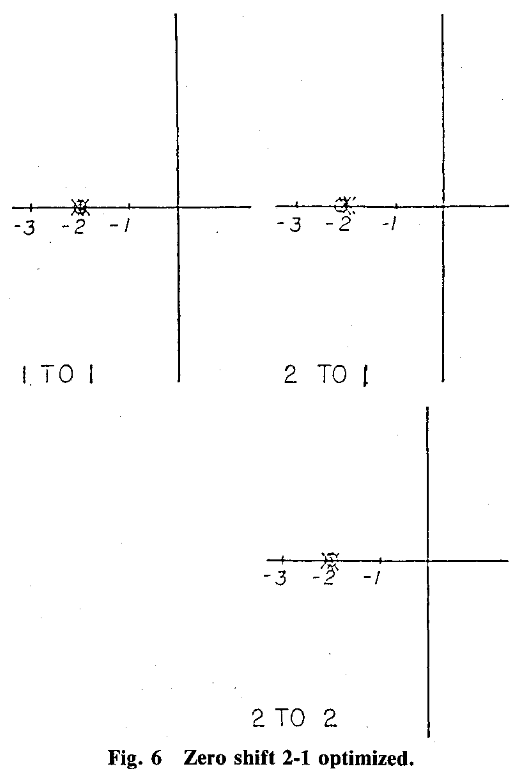

Fig. 6 Zero shift 2-1 optimized.

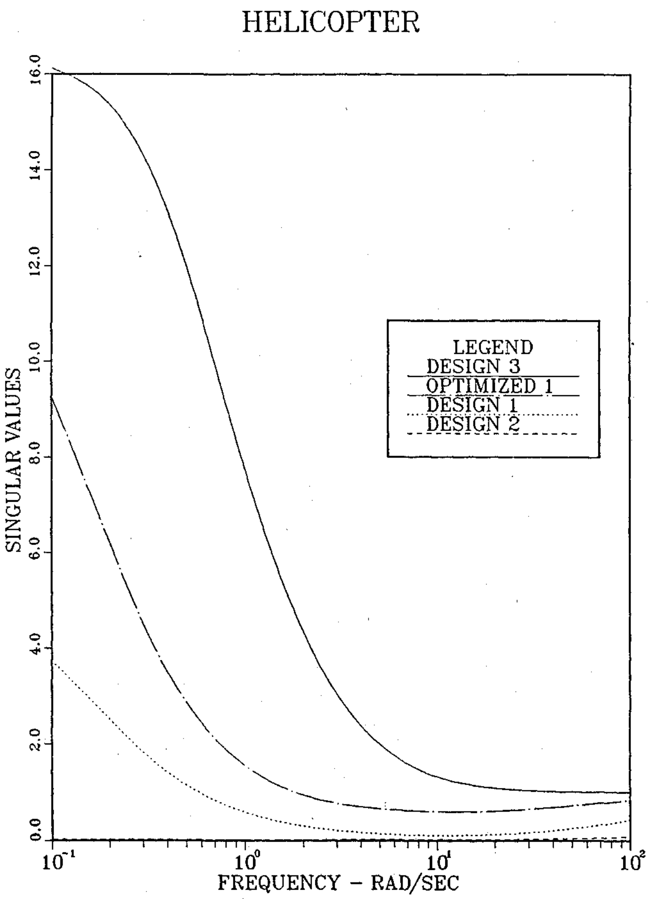

Fig. 7 Singular values.

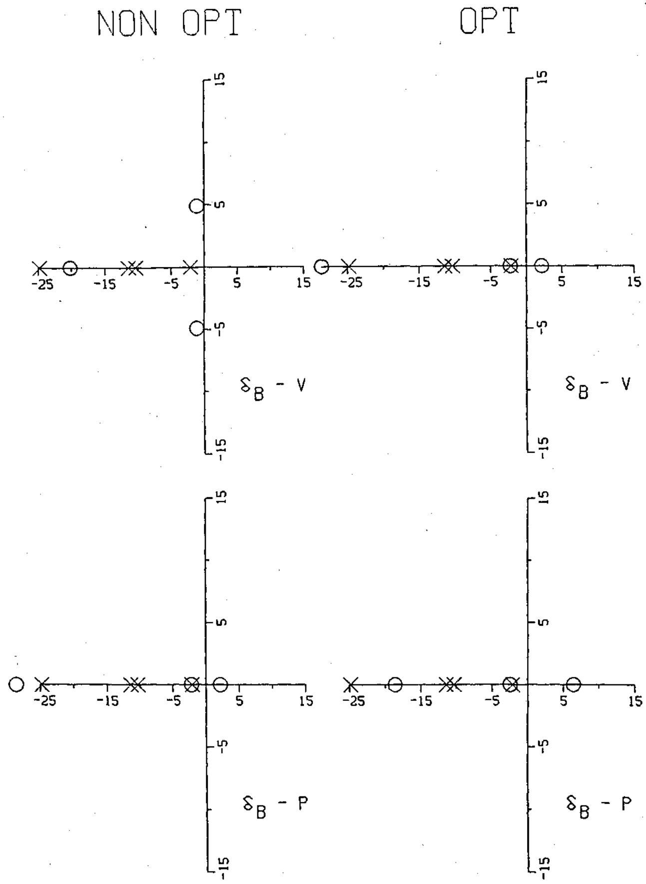

Fig. 9 Zero shifts δ_B helicopter.

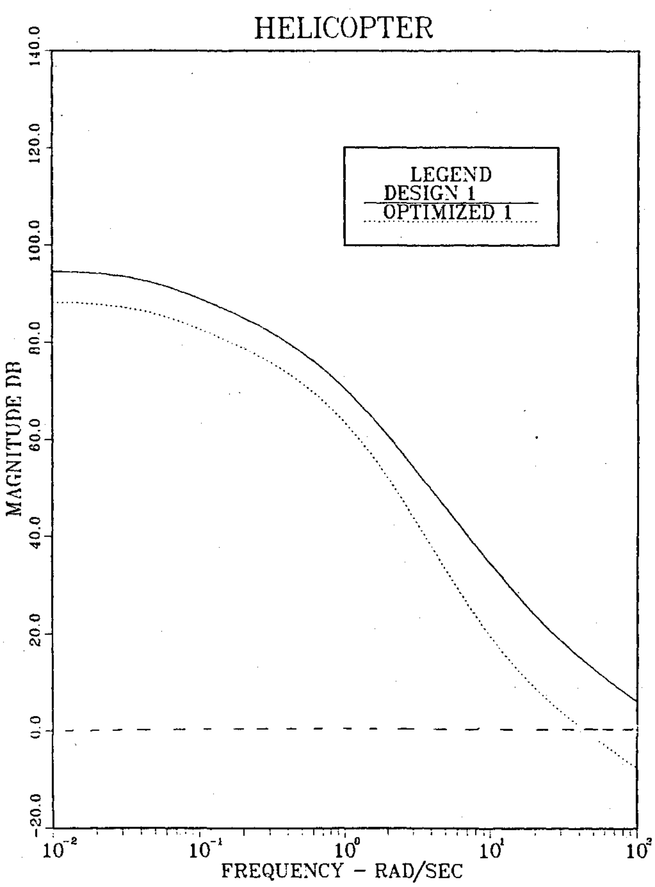

Fig. 8 Loop Bode 2-1.

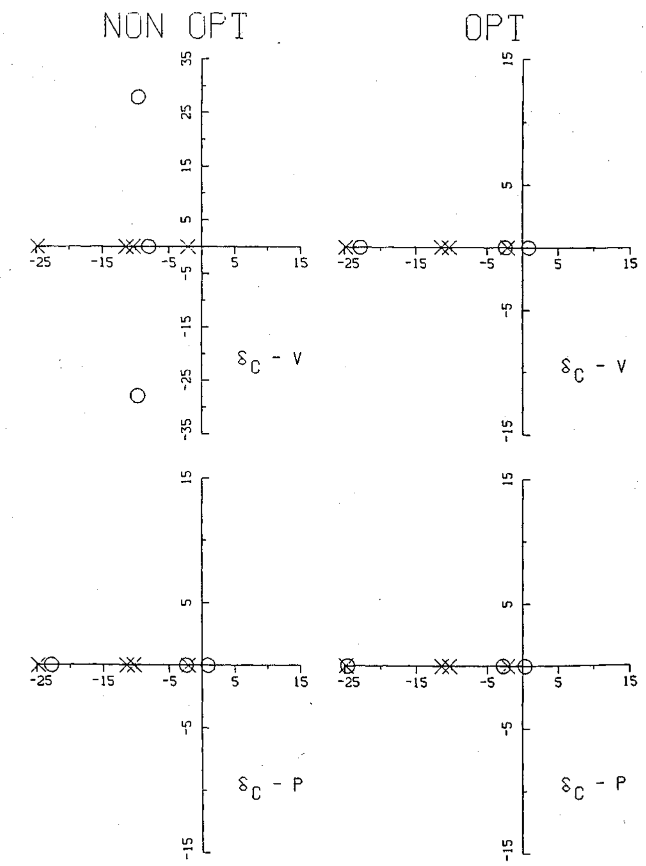

Fig. 10 Zero shifts δ_C helicopter.

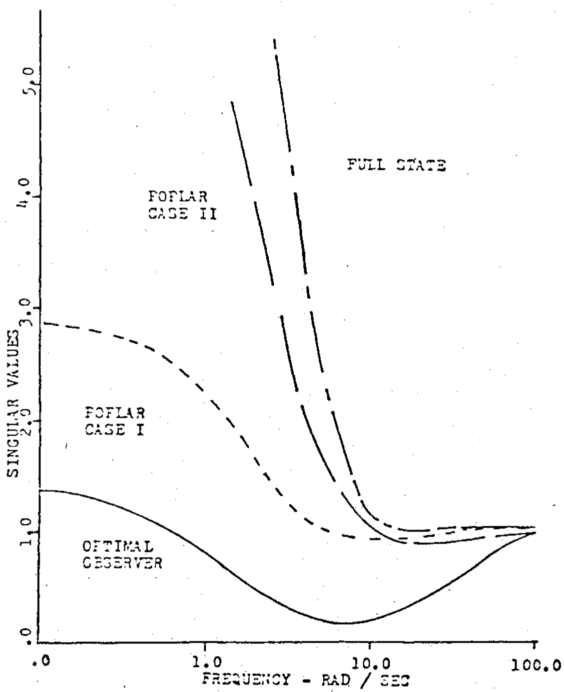

Fig. 11 Observer system singular values.

Conclusion

The pole-placement and robustness design routine, coupled with the Automated Design Synthesis program, has demonstrated the capability to provide designs that solve problems caused by cross-coupling perturbations which reduce robustness. The design improvement is accomplished by modifying the system feedback gains so that the gains in channels affected by cross-coupling are equalized with the system gains to reduce cross-coupling effects. The gain changes are accomplished by zero shifts, which also influence the gain distribution and frequency response of the system. Perturbation problems in multivariable systems have been shown to be detectable by singular value analysis and by Bode magnitude diagrams of the modified open-loop transfer functions of the system. In the transfer function plots, large differentials in Bode gain and bandwidth indicate problem areas for cross-coupling perturbations. Robustness is obtained by modifying those gains and bandwidths associated with the cross-coupling perturbations, thus reducing the energy coupled from the perturbation into other channels. An associated shift of zeros toward poles located in the vicinity of the frequency of the minimum singular value, which tends to equalize the frequency response curve, has been noted.

The use of numerical optimization to recover robustness in observer-based designs was demonstrated. The pole placement and robustness routine was applied to problems previously solved, using the fictitious noise procedure for robustness recovery. The direct manipulation of feedback and filter gains by the pole placement and robustness routine provided a highly robust design with relatively low filter gains. The problem of robustness recovery in filter-observer designs has been solved in a straightforward and highly practical manner.

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